A Dusty Discipline

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How does one see a mathematical idea? Can it be heard, touched, or smelled? If you spend enough time around mathematicians in the heat of research, you tend to believe more and more that when it comes to mathematics, the materials matter. To many, mathematical ideas look and sound and feel and smell a lot like a stick of chalk slapping and then gliding along a blackboard, kicking up plumes of dust as it traces formulas, diagrams, and other mathematical tokens.

Chalk and blackboards first made their mark in higher education at elite military schools, such as the École Polytechnique in France and West Point in the United States, around 1800. Decades of war and geopolitical turmoil, combined with sweeping changes to the scale and social organization of governments, put a new premium on training large corps of elite civil and military engineers. Mathematics was their essential tool, and would also become a gateway subject for efficiently sorting the best and brightest. Blackboards offered instructors a way of working quickly and visibly in front of the large groups of students who would now need to know mathematics to a greater degree than ever before. They also furnished settings of discipline, both literal and figurative, allowing those instructors to examine and correct the work of many students at once or in succession as they solved problems at the board.

In the two intervening centuries, the importance of chalk and blackboards for advanced mathematics grew and grew. Blackboards reigned as the dominant medium of teaching and lecturing for most of the twentieth century, and continue to be an iconic presence in countless settings where mathematics is learned, challenged, and developed anew. As, indeed, it routinely is. While schoolbook mathematics seems as though it has been settled since time immemorial (it has not been, but that is another story), the mathematics taking place in universities and research institutes is changing at a faster rate than ever before. New theorems and results emerge across the world at such a dizzying pace that even the brightest mathematicians sometimes struggle to keep up with breakthroughs in their own and nearby fields of study. Long gone are the days when a single mathematician could even pretend to have a command of the latest ideas of the entire discipline.

The problem of keeping up might seem to lead one towards high technology, but to a surprising extent it leads back to the blackboard. When Barany followed the day-to-day activities of a group of university mathematicians, he found the blackboard was most prominent in their weekly seminar, when they gather after lunch to hear a local or invited colleague's hourlong presentation on the fruits and conundrums of recent and ongoing work. But blackboards are also present in offices, and even the departmental tea room. Their most frequent use came when mathematicians return to the seminar or other rooms to teach mathematics to students, just as their predecessors did two hundred years ago. Wherever they are, blackboards serve as stages for learning, sharing, and discussing mathematics.

Blackboards are still more pervasive when one searches for them in unexpected places. The archetypal blackboard is a large rectangular slab of dark gray slate mounted on a wall, but over their history most blackboards have been made of other materials, some of which are not even black. (This includes the dark green boards in the seminar room of Barany's subjects.) Characteristics of blackboard writing can be found in the pen-and-paper notes researchers scribble for themselves or jot for colleagues. Gestures and ways of referring to ideas in front of a board translate readily to other locations. The blackboard is one part writing surface and two parts state of mind. To understand the blackboard, we realized, is to understand far more about mathematics than just seminars, lectures, and the occasional chalk marking in an office.

Even as blank slates, blackboards are laden with meaning. Because they are large and mostly immobile they greatly affect how other features of offices or seminar rooms can be arranged. Entering a seminar room, one knows where to sit and look even when the speaker has not yet arrived, and the same principle holds for the different kinds of situations present in offices. We noted that when one arrangement of desks and chairs did not quite work it was the desks and chairs, rather than the blackboard, that were rearranged. Staring blankly at its potential users, a blackboard promises a space for writing and discussion. Depending on the context, too much writing on the board prompted users to use an eraser long before anyone intended to use the newly cleared space. Having a blank space available at just the right moment was important enough that mathematicians anticipated the need far in advance, trading present inconvenience for future chalk-based possibilities.

When blackboards are in use, more features come into play. They are big and available: large expanses of board are visible and markable at each point in a presentation, and even the comparatively small boards in researchers' offices are valued for their relative girth. Blackboards are visually shared: users see blackboard marks in largely the same way at the same time. They are slow and loud: the deliberate tapping and sliding of blackboard writing slows users down and makes it difficult to write and talk at the same time, thereby shaping the kinds of descriptions possible at the board. As anyone who has fussed with a video projector or struggled with a dry-erase marker that was a bit too dry appreciates, blackboards are robust and reliable, with very simple means of adding or removing images.

As surfaces, blackboards do more than host writing. They provide the backdrop for the waves, pinches, and swipes with which mathematicians use their hands to illustrate mathematical objects and principles. They also fix ideas to locations, so that instead of having to re-describe a detailed idea from earlier in the talk a lecturer can simply gesture at the location of the chalk writing that corresponded to the prior exposition. We were surprised to find that such gestures are used and seem to work whether or not the chalk writing had been erased in the interim—although sometimes the speaker had to pause after finding that the relevant expression was no longer where it was expected to be.

In addition to these narrative uses in a lecture, locations on the blackboard can have a specifically mathematical significance. Mathematical arguments often involve substituting symbolic expressions for one another, and on the board this can be done by smudging out the old expression and writing the new one in the now-cloudy space where the old one had been. This ability to create continuity between old and new symbols is so important in many cases that speakers frequently will struggle to squeeze the new terms in the too-small space left by the old ones rather than rewrite the whole formula, even when the latter approach would have been substantially easier to read. Boards are also large enough to let the speaker create exaggerated spaces between different parts of a formula, permitting the speaker to stress their conceptual distinctness or to leave room for substitutions and transformations.

And what of the chalk marks themselves? One rarely thinks of what *cannot* be written with chalk, a tool that promises the ability to add and remove marks from a board almost at will. The chalk's shape, its lack of a sharp point, and the angle and force with which it must be applied to make an impression, all conspire to make certain kinds of writing impossible or impractical. Small characters and minute details prove difficult, and it is hard to differentiate scripts or weights in chalk text. Board-users thus resort to large (sometimes abbreviated) marks, borrow typewriter conventions such as underlining or overlining, or employ board-specific notations such as "blackboard bold" characters to denote certain classes of mathematical objects.

Not every trouble has a work-around. Similar to a ball-point pen or pencil on paper, chalk must be dragged along the board's surface to leave a trace. Entrenched mathematical conventions from the era of fountain pens, such as "dotting" a letter to indicate a function's derivative, stymie even experienced lecturers by forcing them to choose between a recognizable dotting gesture and the comparatively cumbersome strokes necessary to leave a visible dot on the board.

These practical considerations have profound effects on how mathematics is done and understood. For one, blackboard writing does not move very well. This means that whenever one makes an argument at a blackboard one must reproduce each step of the argument at the board from scratch. Nothing is pre-written, and (in the ideal of mathematical argument) nothing is pre-given as true. Proofs and constructions proceed step by step, and can be challenged by the audience at each point. It is not possible in a rigorous mathematical presentation, unlike in other disciplines, to drop a mountain of data in front of an interlocutor and then move straight to one's interpretations and conclusions.

In a blackboard lecture, those taking notes write along with the speaker. In a classroom setting, lecturers can expect that most of their board writing will be transcribed with little further annotation. Fewer audience members take notes during a seminar, but the expectation of transcription persists. In particular, what is written on the chalkboard is, in a good lecture, largely self-contained. The division between the speaker's writing and speech parallels a similar division in any mathematical argument. Such arguments combine commentary and explanation (like the presenter's speech) with a rigorous formal exposition that, mathematically, is supposed to stand on its own (like the presenter's writing), even if it might be difficult to understand without the commentary.

Going along step by step with an argument produced at a blackboard gives mathematicians the chance to break an argument that can be extremely difficult to comprehend globally into smaller steps that are possible to understand for many in the audience. This local-but-not-global way of viewing colleagues' work is indispensable in a discipline as vast and quickly changing as mathematics. For while the basic steps of mathematical arguments are often shared among specialists in related areas, the nuances and particularities of a single mathematician's work can be opaque even to recent collaborators. A proof may be true or valid universally, but mathematicians must make sense of it in their own particular ways. So blackboards offer a means of communication in both the obvious sense—as things on which to write—and a more subtle sense in terms of a step-by-step method of exposition.

The objects of that exposition also have certain features enforced by blackboard writing. Any writing on the board can be corrected, annotated, or erased at the board user's will. It is common to see lecturers amend statements as new information becomes relevant, often after a query from the audience. The blackboard lets speakers make those amendments without a messy trail of scribbles or crossings-out, preserving the visual integrity of the record that remains on the board. In this way, speaker and audience alike can believe that the ultimate mathematical objects and statements under consideration maintain a certain conceptual integrity despite all the messy writing and re-writing needed to understand and convey them. This view is a key part of mathematical Platonism, which contends that mathematical objects and truths exist independent of human activity, and represents a central position in the philosophy of mathematics, albeit with many variations.

This principle extends into research environments as well. We noted that blackboards were particularly valuable as surfaces for working out complicated expressions, where it was necessary to array many symbols and images in a setting where they could be viewed, revised, and manipulated. Here, the board's necessity for dealing with complex mathematics shows a genuine practical limit to mathematical comprehension. If the board proves necessary to make some computations comprehensible, then those for which even the board is inadequate are doubly barred from any hope of being shared and understood in a broad community of mathematicians. It is said among mathematicians that the most profound results have a clear and simple statement. The blackboard forces us to recognize the converse: no result, no matter how well it comports to some logical standard of truth, can be accepted if there is no clear way to write and share it.

Hence a striking paradox: the conceptual development of mathematics, apparently the most abstract of disciplines, may be influenced profoundly by the technologies of writing, and even by the mundane physicality of blackboards and chalk.

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